INSTITUTE OF MATHEMATICS, POLISH ACADEMY OF SCIENCES

DISSERTATIONES MATHEMATICAE

Founded in 1952 as ROZPRAWY MATEMATYCZNE
by **Karol Borsuk**

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Sur les courbes hyperelliptiques cyclotomiques et les équations $x^p - y^p = cz^2$

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TABLE DES MATIÈRES

Abstract

Let p be a prime number ≥ 11 and c be a square-free integer ≥ 3. In this paper, we study the diophantine equation $x^p - y^p = cz^2$ in the case where p belongs to $\{11, 13, 17\}$. More precisely, we prove that for those primes, there is no integer solution (x, y, z) to this equation satisfying $\gcd(x, y, z) = 1$ and $xyz \neq 0$ if the integer c has the following property: if ℓ is a prime number dividing c then $\ell \not\equiv 1 \bmod p$. To obtain this result, we consider the hyperelliptic curves $C_p : y^2 = \Phi_p(x)$ and $D_p : py^2 = \Phi_p(x)$, where Φ_p is the pth cyclotomic polynomial, and we determine the sets $C_p(\mathbb{Q})$ and $D_p(\mathbb{Q})$. Using the elliptic Chabauty method, we prove that $C_p(\mathbb{Q}) = \{(-1, -1), (-1, 1), (0, -1), (0, 1)\}$ and $D_p(\mathbb{Q}) = \{(1, -1), (1, 1)\}$ for $p \in \{11, 13, 17\}$.

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