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Lambert series and Liouville's identities

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CONTENTS

Abstract

The relationship between Liouville's arithmetic identities and products of Lambert series is investigated. For example it is shown that Liouville's arithmetic formula for the sum

$$\sum_{\substack{(a,b,x,y)\in\mathbb{N}^4\\ ax+by=n}} (F(a-b)-F(a+b)),$$

where $n\in\mathbb{N}$ and $F:\mathbb{Z}\to\mathbb{C}$ is an even function, is equivalent to the Lambert series for

$$\left(\sum_{n=1}^{\infty}\frac{q^n}{1-q^n}\sin n\theta\right)^2 \qquad (\theta\in\mathbb{R},\ |q|<1)$$

given by Ramanujan.

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